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Market risk based capital for Brazilian insurance companies: A stochastic approach☆

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ABSTRACT

Solvency II defines minimum capital requirements from insurance companies, due to their exposure to risk. Regulatory bodies of the Brazilian insurance market issued regulations based on a deterministic model for the calculation of risk based capital. In this study, we discuss a simple alternative stochastic model that may be more adequate to measure the minimum capital, based on an asset and liability management mechanism. By allowing the estimation of a probability distribution of losses, the model is useful to identify not only average results but also extreme results for the portfolio. Focusing on the market risk associated with interest rate exposures, we apply a stochastic model to investigate risks from a pension plan product of an insurance company operating in Brazil. The Brazilian case is relevant, since due to its high interest rate levels, high interest volatility and the usual investment in inflation–index bonds, deterministic models can poorly reflect the effective risk exposure of insurers. For the analysis of the actuarial liabilities, we use demographic data from the insurer. For the analysis of the assets, we investigate different hypothetical portfolios of investments in government bonds, analyzing how interest rates changes and indexes mismatch impact the potential loss. The results of the simulation of the stochastic model, for all scenarios, show potential losses that are substantially higher than the values established by the regulatory model. Estimate of potential loss can reach differences of up to 419%. The results suggest that the deterministic model defined by the regulatory agencies may be a fragile reference to the real capital needs of an insurance company, especially when considering the economic environment of emerging countries.

1. Introduction

The National Council of Private Insurance (*CNSP*) and the Superintendency of Private Insurance (*SUSEP*), regulatory bodies of the Brazilian insurance market, have established risk-based capital assessment mechanisms for the solvency of insurance companies, following *Solvency II* guidelines. Aiming to promote greater safety of the insurance industry, notably to prevent systemic crises, *Solvency II* represents a series of standards for insurance and reinsurance operations, notably in the international market. Similarly to the *Basel II* guidelines applicable to the banking system, *Solvency II* is based on the principle that insurance institutions must have sufficient capital to cope with potential losses in their operations, thereby avoiding that financial difficulties of individual risk could

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disseminate to the market as a whole.

Considering the focus of *Solvency II* on the international market, *SUSEP* and *CNSP* have sought to make adaptations in the guidelines in order to make regulatory models more adequate to the context of the Brazilian insurance market. However, although adjustments could be necessary, given the high levels of interest rates and the high volatility of the country's economic environment, it is argued that the model indicated by Brazilian regulatory agencies may be fragile.

In particular, the calculation of market risk capital may not necessarily reflect the true exposure of the insurers to risk arising from their operations. Since the insurance market in Brazil has not gone through stressed conditions, even during the 2007–2008 global financial crisis, it is not clear whether the regulatory agencies provide a safe capital requirement for insurers.

The objective of this study is to discuss a methodology of analysis of capital requirements that more realistically reflects the exposure to which an insurer is exposed, considering particularly a high volatile economic environment. Through simulations based on actual data from an insurer with operations in Brazil, the results generated by the stochastic model are compared with those of the regulatory model based on a deterministic approach, taking into account a product similar to a participating life insurance. Management decisions about asset investment or portfolio selection (Eckert, Gatzert, & Martin, 2016; Gollier and Wibaut, 1992) in a participating life insurance can substantially impact the insurer's exposure to risk. Therefore, market risk analysis of the investment portfolio of insurers becomes relevant to the solvency of the insurance market.

From the point of view of regulators, deterministic approaches are usually based on simple calculation of capital requirements, but at the same time, should be conservative. Since deterministic models do not incorporate probabilistic stress scenarios, requirements may be established at exaggerated levels to avoid financial distress due to unanticipated events. Therefore, by being conservative, a regulatory deterministic model will not necessarily lead to a lower capital requirement than internal models based on a stochastic approach.

One can argue that, by taking into account extreme events, stochastic models could naturally lead to measures of higher risk exposure. However, comparing deterministic regulatory requirements with a more realistic probabilistic approach is still relevant, especially to assess the magnitude of the difference of risk assessment. More specifically, if differences are substantial, regulatory guidelines based on deterministic and simplistic models are underestimating the real risk level of the insurance industry. In addition, even if insurance companies would be allowed to choose more accurate, internal risk models, they would not be willing to adopt stochastic models due to a higher capital allocation.

Using Monte Carlo simulation techniques, the study comparatively evaluates results of capital requirements from the stochastic model with the results generated using the formula defined by the Brazilian regulator. Studies on the life insurance market using simulation are relatively frequent, such as Faust, Schmeiser, and Zemp (2012) for performance analysis of participating life insurance, Zaglauer and Bauer (2008) for the pricing of participating life insurance contracts considering stochastic interest rates, and Bohnert and Gatzert (2012) that analyzes surplus appropriation schemes. However, the study of highly unstable markets is not usual. More specifically, the Brazilian economic environment presents high base interest rates as well as high volatility of interest rates and therefore should be a relevant environment to be studied by risk-bearing institutions.

Although interest rates are high, allowing financial assets to have a high return, when compared to the actuarial liability, the high market volatility implies that gaps between asset and liability cash flows can generate large losses. In addition, our findings indicate that the regulatory model of the Brazilian market leads to underestimated market risk requirements for an insurer's portfolio. This result implies a fragility of the insurance segment, reflecting the need for a more thorough discussion of the Brazilian regulatory model for *Solvency II*. The results of our study suggest that, even with a stable actuarial liability, insurers may face solvency problems simply because they operate in a lower capital requirement environment than would be necessary to face their real exposure to market risk.

More specifically, the simulations allow not only the identification of mean values but also unlikely, yet possible, values, which are important elements for risk analysis, especially in highly volatile environments such as Brazil. Finally, it is important to highlight that studies for specific insurance markets were explored by other works such as Eling and Holder (2013) in Germany, Jevtić and Regis (2015) with data from the United Kingdom.

The paper is structured as follows. In the next section, we present a brief review of themes associated with risk capital in the insurance industry. Then, we describe the methodology and data used to estimate the parameters for the simulations. We present the simulation results, comparing values from the stochastic model with data from the model suggested by the regulator. Finally, some implications of the stochastic model are discussed, showing advantages and limitations.

2. Theoretical background

An insurance company has as its main operation the sale of a product that makes it possible to transfer the risk of a party, the insured, to a counterparty, the insurer, upon payment of a premium value. According to Murphy (2011), the insurance allows sharing financial losses arising from the occurrence of an undesirable event. Thus, instead of an individual face the total loss of a possible claim, he can dilute at least his financial misfortune with all policyholders, by paying the insurance premium.

It is important to emphasize that risk transfer is not only linked to the existence of a material good. Operations associated with random rents from death, for example, also involve a risk transfer in exchange for premium payments. Thus, an individual can transfer, at least in financial terms, the risk of survival to an insurance company, disbursing premiums, so as to accumulate a technical reserve until the time of retirement, when he begins to obtain monetary benefits.

In this insurance product, the participant wishes to transfer the risk of financial loss, due to the survival, reducing his exposure in relation to the accumulation of a reserve that is insufficient to cover his expenses after retirement. Thus, when contracting this type of insurance, the participant has an actuarial benefit based on financial payments made by the insurer, which in turn will price the periodic contributions based on mortality tables adhering to the characteristics of the insured population (Jordan, 1991). Although this product is mainly acquired by individuals, it could be also paid by firms to their employees, such as discussed by Atanasova and Hrazdil (2010).

In addition to the risk of survival described earlier, there is also the risk of the insured dies or becomes invalid. In the case of a claim, that is, the death or permanent disability of the participant, a designated beneficiary receives payments, allowing the family to have a lower economic impact due to the misfortune.

It should be noted that the insurer, taking into account its obligation, has an actuarial liability that depends on the death, disability or survival of the insured. The investment of the premium, payed by the insured, in profitable assets should generate future cash flows that may meet the actuarial liability of the insurer. In this context, market risk, which arises from the potential fluctuation of the value of the portfolio of assets due to market parameters such as interest rates, exchange rates, stock or commodities prices, becomes relevant to the solvency of an insurer.

Life insurance with death benefits under *Solvency II* is a relevant topic in the financial literature. For instance, Meyricke and Sherris (2014) study hedging strategies against longevity risks for life insurers and Cheng and Li (2018) investigate early default risk and surrender risk. Hainaut, Devolder, and Pelsser (2018) develop a robust framework to define solvency capital requirements for participating life insurance.

Several studies analyze impacts of *Solvency II* on the insurance companies, especially on asset allocation, portfolio optimization and risk management. For instance, Arias, Foulquier, and Maistre (2012) focus on bond management under *Solvency II*, studying risk measurement, sensitivity analysis and asset allocation. Braun, Schmeiser, and Schreiber (2015) assess constraints from regulation in portfolio optimization in the insurance industry. Considering a specific country regulation, van Bragt, Steehouwer, and Waalwijk (2010) study an ALM model for the Dutch regulatory framework. We assess the Brazilian regulatory context of market risk, with emphasis on interest rate risk. It is important to highlight that interest rate risk, spread risk and equity risk are the main components of market risk, within the solvency capital requirements in the European industry (EIOPA, 2011).

The method to calculate risk capital based on market risk suggested by the European regulator considers segregation in several modules. Each of these modules is defined after the analysis of the main risk factors that expose insurance companies to the likelihood of losses. The following risk factors are taken into account: interest rate, equity, properties, foreign currency, credit spread, concentration and liquidity premium. Following the isolated assessment for each risk factor, the aggregation of risk is performed considering correlations among risk factors.

The model adopted by *SUSEP*, the Brazilian regulator, considers the application of predefined factors on the net exposure, resulting from the difference between assets and liabilities within certain time buckets, divided by vertices. These factors are determined by the regulator and the methodology considers market risk factors that expose insurance companies to potential losses, similarly to the European model. In the Brazilian model, the following risk factors are considered: pre and post fixed interest rates, equity, exchange rate and commodities.

Therefore, regulatory models were defined considering the risk factors that the insurance companies of each region would be more exposed to. The main difference between the models adopted by SUSEP and the European regulator is related to the calculation of risk capital based on market risk. The European model considers the calculation for each risk factor in isolation and, later, aggregate risk is obtained through the correlation among risk factor. In contrast, the Brazilian model considers the application of predefined factors in net exposures, obtaining the value of risk capital based on market risk directly, without the need to aggregate risks using correlation among risk factors.

In Brazil, *SUSEP* regulates the need for insurance companies to maintain a minimum equity capital based on exposure to market risk. According to Sandström (2010), a solvent institution must have a surplus of assets in relation to its liabilities, aiming to honor its commitments over time. The requirement for solvency levels is relevant to guarantee the rights of policyholders and beneficiaries and to generate value for insurer's own management and stockholders through business continuity (Pentikäinen, 1967).

The model for required minimum capital by the Brazilian regulatory agency follows the rationale discussed by Hickman, Cody, Maynard, Trowbridge, and Turner (1979), which considers the expected net present value of the future cash flows of assets and liabilities. More specifically, in the *SUSEP*'s model, three relevant risks are associated with capital requirements: credit risk, linked to creditworthiness of assets; underwriting risk, due to possible pricing problems of the insurance premium; and market risk, linked, in the case of insurance company portfolios in Brazil, mainly to interest rate fluctuations.

The equity capital required by the Brazilian regulatory agency involves the calculation of exposures to credit, underwriting and market risk, considering the correlations between risk factors, plus a value due to operational risk. We should emphasized that *Solvency II* also addresses liquidity risk that, in the Brazilian case, does not involve an additional capital requirement, but rather the compliance with a liquidity ratio based on the minimum capital and the amount of liquid assets. The required capital, according to (Bhatia, 2009), can be seen as a cushion that allows the absorption of unexpected losses, giving greater confidence to investors and customers.

The focus of our study involves the market risk arising from the existence of potential fluctuations on interest rates. The *SUSEP*'s model follows European Commission studies that promote the discussion of a solvency structure in the insurance market, called *Solvency II* (EC, 2006), adapted from the three pillars of *Basel II*, which is applied to the banking sector. In Pillar I, guidelines on capital requirements are presented. In Pillar II, supervisory procedures are discussed and, and in Pillar III, reports on market discipline and information disclosure are defined (Bryce, Webb, Cheevers, Ring, & Clark, 2016).

Regarding specifically to market risk, the basis for the calculation formula suggested by the European regulatory body is the ΔNAV, given by the variation of the difference between asset and liability values. This calculation takes into account changes in the term structure of interest rates, using two different scenarios for each maturity, and multiplying the Δ NAVs by calibration factors (CEIOPS, 2009; Comission, 2015).

In our simulations, we evaluate the sensitivity of present values to interest rates that are stressed at the level of a 99.5% *VaR*, using linear interpolation of rates for nonexistent maturities (CEIOPS, 2010b). Finally, the higher result of ΔNAV represents a stress of interest rate risk. Analogous procedure is conducted for volatility (CEIOPS, 2009).

Within market risk, in situations of currency mismatch, the structure used is analogous. One obtain ΔNAV in different exchange rate scenarios for assets or liabilities. The analysis is performed independently for each currency to which the investment portfolio is exposed (CEIOPS, 2009). Within the scope of market risk, one can also consider, for instance, spread risk, equity risk and concentration risk (CEIOPS, 2009, 2010a; Comission, 2015). In the context of this work, taking into account the analysis of assets and liabilities in Brazilian currency (*BRL*), given the characteristics of the insurer's investment portfolio, market risk will be reflected only by the exposure to potential fluctuations of interest rates and inflation indexes that adjust cash flows from assets and liabilities.

This study seeks to present the advantages of a stochastic model over the Brazilian regulatory requirement, aiming to investigate situations that are closer to reality. By using a parameterization that allows for updates from current market data, the stochastic model based on simulations allows not only a faster reflection of changes in the structures of interest rates and inflation indexes but also a more thorough analysis of extreme market situations.

3. Method

3.1. Demographic data and indexation of assets and liabilities

After presenting a brief context on risk in the insurance segment, we describe the commonly used procedures for calculating actuarial liabilities using dynamic mortality tables. Additionally, for valuation of marked-to-market asset prices, we discuss a stochastic model focused on the diffusion process of interest rates and indexes that adjust cash flows of government bonds. It is important to emphasize that, due to the high volatile economic environment in Brazil, inflation-indexed bonds are usually included in insurer's asset portfolios.

To conduct the quantitative analysis, we use real data from a pension plan of an insurance company operating in the Brazilian market. Taking into account the confidentiality of information, data is modified by a fixed factor, known only by the researchers, making it impossible to readers identify the original data or the financial institution. Estimates of parameters of the stochastic model are obtained from the modified data. This adjustment does not compromise the study, since the objective is to analyze the application of a model to handle potential mismatches between the asset portfolio and the actuarial liabilities.

Statistics collected from the insurer database indicate that 55% of the participants are men and 45% of the participants are women, with a mean age of 50 years and standard deviation of 7 years. All participants have spouses, with an average of 45 years of age and standard deviation of 5 years. Information on age as well as on the annual benefit amount are presented in Table 1 and serves as parameter estimates of the probability distributions used in the simulations.

Demographic data is used to forecast the actuarial liability cash flows. In the model, segregation of participants according to gender and spouse status was considered relevant. For the forecast mechanism, in relation to the age of both the participants and the spouses, we assume that the probability distribution is truncated Gaussian, with ages greater than zero, and parameters described according to those presented in the previous paragraph. The same procedure is used to determine the value of the annual benefit for the individuals, establishing that this variable also has a truncated Gaussian distribution, assuming only positive values, with parameters equivalent to the values in Table 1, estimated through data collected from the insurer.

For the analysis of the portfolio of assets associated with the pension fund, we follow a process similar to the demographic data study, using real data to establish the investments. However, due to confidentiality issues, we do not exactly replicate the insurer's portfolio. For this study, we build a portfolio that includes financial products usually traded by insurance companies in Brazil, more particularly fixed-rate securities issued by the federal government, which have very low credit risk. As the base interested rate in Brazil is usually high, insurance companies prefer concentrating the asset portfolio in federal bonds. In this paper, three types of bonds issued by the government are used: National Treasury Notes – Series B (*NTN–B*), National Treasury Notes – Series C (*NTN—C*) and Financial Treasury Bills (*LFT*), with various maturities.

The *NTN–B* and *NTN–C*, notably long–term investments, have returns linked to the variation of inflation indexes: the National Consumer Price Index (*IPCA*) and the General Market Price Index (*IGP–M*), respectively. There is also a minimum interest rate defined at the time of acquisition of the bond. The holder of these securities receives semiannual income based on the minimum interest rate plus the pertinent inflation index, and the face value of the bond at maturity. The *LFT*, considered very short–term investment, follows the daily variation of the rate of the Special System of Settlement and Custody (*SELIC*), which represents the base rate of the

Table 1
Parameters associated with actuarial liabilities.

Field	Mean	Standard deviation
Age of participant	50	7
Age of spouse	45	5
Annual benefit amount	22,000.00	12,347.13

Table 2
Mismatch indexes scenarios for assets and liabilities.

Scenario	Index	
	Asset	Liability
1	IPCA	IPCA
2	IGPM	IGPM
3	IGPM	IPCA
4	IPCA	IGPM
5	50% IPCA 50% IGP–M	IPCA
6	50% IPCA 50% IGP–M	IGP–M

Brazilian economy. The holder of the *LTF* receives the interest and principal on the maturity of the paper.

Our study analyzed different mismatch scenarios in assets and liabilities, considering different combinations of assets, described in Table 2. Using this method, we can investigate the sensitivity of risk capital in relation to the mismatch of indexes.

In the first scenario, the hypothetical composition of the investment portfolio includes only *NTN–B* and *LFT* securities, with different maturities, whereas the *NTN–C* was used in the analysis of the other scenarios. For instance, the composition of the insurance company in scenario 1 is presented in Table 3. Each scenario has analogous compositions aiming to reflect the mismatch of indexes.

3.2. Forecast of actuarial liability cash flows

The actuarial cash flows of an insurer's liabilities reflect the disbursements required to meet obligations over time with the insureds. The cash flows are projected taking into account the current and future beneficiaries, the demographic aspects related to the survival of the individuals, the coverage values of the contracted benefit, the duration of the benefits and the eventual fluctuations of the interest rates.

Therefore, the forecast of the liability cash flows has, in addition to a financial component, actuarial elements associated with the probability of survival of an individual. The survival functions are decreasing, with probability equal to 1 at the initial age and 0 at the maximum age, usually established at 115 years. For this study we used the biometric table *AT 2000* segregated by gender, relative to the North American population, containing the probabilities of death within a year. It is important to highlight that, due to the lack of a thorough study of mortality in Brazil, insurers usually use the data available in the North American *AT 2000* information. It is important to highlight that *AT-2000* table is used with Brazilian yearly data from mortality rates released by the Brazilian Institute of Geography and Statistics (*IBGE*). We then apply the model developed by Lee and Carter (1992) to transform the static *AT-2000* table into a dynamic one.

From the biometric table, establishing q_x as the probability of an individual dying between the age x and $x + 1$, one can define p_x as the probability of the individual surviving between the ages x and $x + 1$:

$$p_x = 1 - q_x \tag{1}$$

More generally, we can define $p_{x,t}$ as the probability of an individual surviving between ages x ad $x + t$:

$$p_{x,t} = \prod_{i=0}^{t} p_{x+i} \tag{2}$$

The financial analysis of the pension product is based on the discount at, fair value, of the future cash flow using current interest rates. The actuarial analysis is conducted in a similar way to the financial analysis, using interest rates to calculate future or present values. However, the actuarial analysis also incorporates the possibility of occurrence of a random event, the survival of an individual, which affects the cash flows. The adjustment in the cash flows is made weighting the probability of survival of the individual, considering his age.

The actuarial present value a_x can therefore be obtained using the following relation:

Table 3
Composition of the theoretical investment portfolio in scenario 1.

Security	Due date	Quantity	Unit price	Value
National Treasury Notes - Series B	08/15/2017	118,063	2649.94	312,859,051.56
National Treasury Notes - Series B	08/15/2020	314,834	2572.80	810,004,521.22
National Treasury Notes - Series B	08/15/2022	314,834	2576.41	811,141,071.41
National Treasury Notes - Series B	08/15/2024	118,063	2576.69	304,210,959.33
National Treasury Notes - Series B	08/15/2035	118,063	2635.33	311,134,155.63
National Treasury Notes - Series B	08/15/2040	260,279	2597.73	676,135,357.99
Financial Treasury Bills	09/01/2021	55,593	6925.33	385,000,000.00
Total				3,610,485,117.15

$$a_x = \sum_{t=1}^{w-x-1} \frac{1}{(1+i)^t} \cdot p_{x,t} = \sum_{t=1}^{\omega-x-1} \nu^t \cdot p_{x,t} \tag{3}$$

where:

- t represents the number, in years, of the maturity of the cash flow;
- x represents the age, in years, of the individual;
- ω represents the last age of the biometric table;
- i represents the yield rate associated with the maturity of the cash flow;
- ν represents the discount factor; and
- $p_{x,t}$ represents the probability of an individual of age x reaching age $x + t$.

As the pension plan takes also into account the existence of a spouse whose survival also impacts the cash flows, the actuarial calculation must incorporate the probability of survival of another individual. The payment flow is considered in its entirety in the case of survival of the participant and the spouse or in the case of survival only of the participant. In the case of survival of only the spouse, the benefit value suffers a reduction β, referring to a percentage defined in the plan. Without loss of generality, in this study, a 70% reduction was used in relation to the initial benefit, thus representing a pension to the spouse in case of the death of the participant. Deaths of participant and spouse are considered independent.

The actuarial present value of a cash flow that considers the survival of the participant with age x and the spouse with age y is:

$$a_{x,y} = \sum_{t=1}^{w-x-1} \nu^t p_{x,t} p_{y,t} \tag{4}$$

When there is a reduction β of the benefit due to the decease of the participant and survival of the other beneficiary, the actuarial present value APV is given by:

$$APV = a_x + \beta(a_y - a_{x,y}) = \sum_{t=1}^{w-x-1} \nu^t [p_{x,t} + \beta(p_{y,t} - p_{x,t} p_{y,t})] \tag{5}$$

It is important to note that the described formulations allow the analysis of the actuarial present value of cash flows, and can be used to forecast the future actuarial outflow, considering a deterministic model. However, the purpose of this paper is to use a stochastic mechanism for the projection of assets and liabilities, aiming to assess the necessary level of capital requirement associated with market risk. For the liability, the inclusion of the stochastic element is based on the use of dynamic biometric tables, as opposed to the discussion held up to this moment, which considered the table to be static in time.

In a static setting, using the male *AT 2000* table, for example, an individual, who at $t = 0$ ages 50 years, has a probability of death equal to 0.00333. Similarly, another individual, who at time $t = 10$ is 50 years old and therefore, at time $t = 0$ aged 40 years, presents the same probability of death as 0.00333. Therefore, the static tables do not take into account the change in the probability of death of a specific age over the years.

In contrast, the dynamic tables seek to capture the demographic evolution, incorporating the aging trend of the population that, in general, suggests an increasing in life expectancy. This demographic trend is used to define new probabilities of death over time, within a specified age.

For the construction of the dynamic tables, we use the method proposed by Lee and Carter (1992), which allows the prediction of mortality through a demographic model, segregated by age, from the application of time series analysis techniques. The indicator of variation of mortality over time is obtained through the decomposition of mortality matrices. Thus, Lee and Carter (1992) propose a stochastic method of projecting mortality and confidence interval for each age, considering only historical data, given by:

$$\ln(m_{x,t}) = a_x + \beta_x k_t + \epsilon_{x,t} \tag{6}$$

where:

- $m_{x,t}$ represents the central mortality rate for age x in year t;
- α_x, β_x and k_t are parameters of the model;
- $\epsilon_{x,t} \sim N(0, \sigma^2$ represents the random error.

In the model of Lee and Carter (1992), the parameter α_x represents a vector of values specific to each age x, associated with the age-mortality profile, without taking into account the time, while the parameter k_t represents the variation of the level of mortality with time t, incorporating the temporal trend of mortality. The reduction of the k_t parameter over time reflects a reduction in mortality and, consequently, an increase in life expectancy (Lee and Carter, 1992). The parameter β_x describes the changes in mortality rates associated with age, considering the level of mortality rate in t. Other studies, such as Bohnert and Gatzert (2012), use the Lee and Carter (1992), in an extended version, to obtain mortality rates.

For the estimation of the parameters of the dynamic model, a set of mortality tables of the Brazilian population, made available, annually and by gender, by the Brazilian Institute of Geography and Statistics (*IBGE*). These mortality tables reflect the evolution of the population during a time horizon, allowing the extraction of central rates of mortality and their trends and the use of the model suggested by Lee and Carter (1992).

Considering the formulas of the present actuarial values and the model of dynamic biometric tables obtained from the application of the Lee and Carter (1992), we conducted forecasts of the cash flows of the actuarial liability. The yield curve used to discount cash flows also has a stochastic nature. The mechanism used to forecast the term structure of interest rate is similar to that used for the study of the cash flows of the assets described in the following sub-section.

3.3. Forecast of asset cash flows

For the determination of capital based on market risk, an analysis of the insurer's assets is necessary, since the value of the investment portfolio should be greater than the value of the actuarial liabilities. As previously discussed, we use, in this study, a portfolio composed of public bonds, which, within the context of market risk, exposes the investments to the yield curve and to the indexes that adjust the bonds cash flows.

The yield curve represents the term structure of interest rates by maturity, and is used to price cash flow streams that are distributed over time (Fabozzi et al., 2002). Due to the unavailability of bonds with certain maturities, interpolation or extrapolation of interest rate data is necessary to bring all cash flows to present value. The yield curve is a market risk factor, as interest rates may undergo changes and may impact the market value of the asset portfolio.

The market value of the security is obtained by bringing the future cash flows of the bond, considering the coupon defined in the instant of the issuance of the security, to the present value using the current market interest rates, defined by the yield curve. It should be noted that the calculation of the present value of the actuarial liability flows is also impacted by the yield curve.

Another relevant market risk factor for the insurer's assets is linked to the fluctuation of the securities' indexes. The cash inflows of the assets are adjusted by inflation indexes *IPCA* and *IGP–M*, respectively, for the *NTN–B* and *NTN–C*, and may cause a relevant gap in relation to the liabilities outflows, implying an important component of market risk.

In this study, we apply the traditional *CIR* model (Cox, Ingersoll and Ross, 1985), widely used by market practitioners and also incorporated in the study of Russo et al. (2017) for the proposition of a framework for surrender modeling. In addition to allow mean reversion, the *CIR* model also has other advantages over the seminal model of Vasicek (1977), mainly due to preventing the possibility of negative interest rates. The *CIR* model was applied both to the construction of term structure of interest rates that brings the *NTN–B* and *NTN–C* bonds to present value and to the projection of the *SELIC* rate for valuing *LFT* bonds.

The *CIR* model assumes that the interest rate follows a Brownian motion described by the following differential equation:

$$dr = a(b - r)dt + \sigma\sqrt{r}dz \tag{7}$$

where:

- r is the short–term interest rate;
- b is the long–term interest rate;
- a is associated with the speed of mean reversion of the short–term interest rate to the long–term interest rate;
- σ is the volatility of the short–term interest rate.

Once the underlying interest rate model is defined, we use historical market data to estimate the *CIR* parameters and simulate the behavior of current bond rates. We chose to use securities with a long historical data to estimate parameters that reflect the variability of the rate in very different scenarios. Therefore, we try to consider distinct economic cycles making the scenarios generated by the simulations quite broad, aiming to incorporate different behavior of interest rates. Table 4 shows the maturities of the specific securities and the *SELIC* rate that were used in this study.

Parameter estimates of the *CIR* model, using the maximum likelihood method, are given in Table 5 below. Based on these estimates, term structure interest rate can be constructed in different scenarios.

The investments of the insurer in financial securities are exposed not only to the risk of interest rate fluctuation but also to the behavior of the indexes of the securities, *IPCA* and *IGP–M*. For the simulation process, it is therefore necessary to adopt a projection model for these indexes. Historical data were used to estimate the parameters of a distribution that was defined as Gaussian. The values of the estimated parameters are presented in Table 6.

3.4. Determination of the market risk based capital

Once the parameters of the stochastic models are estimated, we proceeded to the effective simulation of the proposed scenarios

Table 4
Data information of historical series of interest rates.

Bond/Rate	Name	Beginning of the studied series	End of the studied series	Period
NTN–B	NTN–B 760199	12/31/2003	06/30/2015	Monthly
NTN–C	NTN–C 770100	12/31/2003	06/30/2015	Monthly
SELIC	–	12/31/2003	06/30/2015	Monthly

Table 5
Estimated parameters for the CIR model.

Interest rate structure	Parameters			Maximum likelihood
	a	b	σ	
NTN–B	0.0325	0.0611	0.0031	601.4979
NTN–B	0.0239	0.0570	0.0024	633.9328
SELIC	0.0120	0.01060	0.0040	567.8007

Table 6
Historical series information and indexes parameters.

Index	Beginning of the studied series	End of the studied series	Period	Parameter	
				M	σ
IPCA	01/01/2007	06/30/2015	Monthly	0.4870%	0.2537%
IGP–M	01/01/2007	06/30/2015	Monthly	0.5095%	0.5668%

and calculation of the market risk based capital.

Differently from the calculation defined by *Solvency II* and *CNSP* regulations, which are based on weighting factors and ΔNAV, the methodology considered in this paper uses a stochastic simulation procedure to determine the net present values of each analyzed scenario. The rationale of the stochastic model is based on the fact that the calculation of several simulations for each scenario replaces the application of weighting factors, obtaining, therefore, a distribution of net present values, instead of a single value in a deterministic approach. This simulation mechanism allows obtaining stressed values, bringing more realistic elements to the process of risk analysis, since extreme potential results can be relevant information to the decision making process. In the model of this study, the actuarial present value APV of the cash flows of the liabilities is given by:

$$APV = \frac{1}{(1+r)^t}(FV_t + FR_t) \tag{8}$$

where:

- FV_t is the actuarial cash flow benefit in t until death;
- FR_t is the actuarial reversible cash flow in t.

Similarly, the formulation of the present value PV of the assets cash flows is presented below:

$$PV = \frac{1}{(1+r)^t}(F_{t,bond} + Cash_t) \tag{9}$$

where:

- $F_{t,bond}$ is the cash flow in t of the government bond (NTN–B or NTN–C) in the insurer's portfolio;
- $Cash_t$ is the cash amount in instant t.

We then consolidate the present value of assets and liabilities cash flows:

$$NPV = APV - PV \tag{10}$$

The value-at-risk (*VaR*) metric was used to calculate the difference between the observed net present value and the stressed net present value, with a level of 99.5%, which is the same percentile defined by *Solvency II*. The difference between the present values was established as risk capital requirement, based on the market risk, from the stochastic model. In order to compare the result of this methodology with the result obtained by the regulatory model, we also implemented *SUSEP*'s methodology using the same asset and liability data.

The analysis of the *SUSEP*'s model follows a prior definition of weighting factors to be applied in each vertex of the cash flows, considering maturity and indexes of assets and liabilities. These factors are determined by *SUSEP*, without explicit disclosure about the algorithm, the database or the history of information used. In this study, the simulations from the proposed stochastic model are compared with the results obtained by applying the *SUSEP*'s model, considering the weighting factors previously established by the regulatory body.

4. Results

We studied the behavior of the cash flows related to the actuarial liability and to the portfolio of investments in assets. Taking into

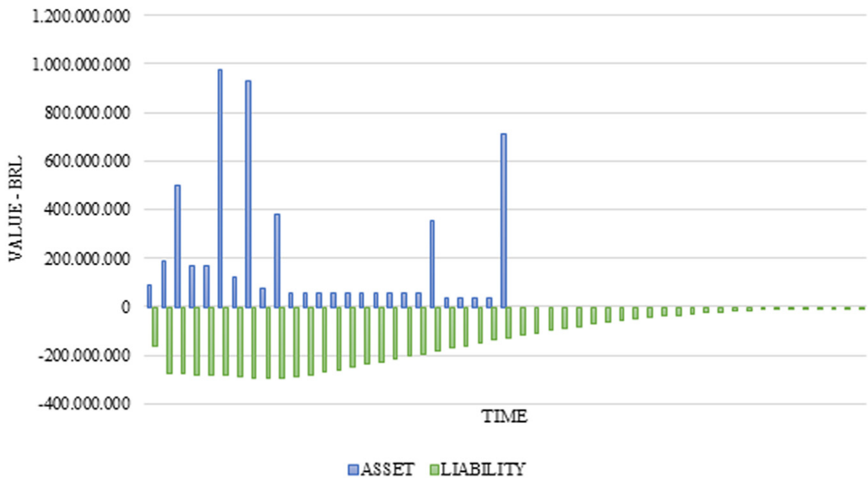

Fig. 1. Mismatch of cash flow for scenario 1 using the stochastic model.

account the assets, for illustrative purposes, we present only the profile of the flows deriving for investments in *NTN–Bs* in scenario 1. Fig. 1 compares the cash flows of assets and liabilities, allowing the identification of possible gaps in the short and long term cash flows.

We present the results of market risk capital according to the stochastic model, comparing them with the results of the regulatory model. As shown in Table 2, scenario 1 considered the allocation of the total portfolio in *NTN–B* and the liabilities adjusted by the *IPCA*. In scenario 2, we considered the assets of the portfolio invested in *NTN–C* and the index of liabilities in *IGP–M*. In scenarios 3 and 4, we considered gaps between asset and liability indexes. Scenario 3 considers the allocation of the asset portfolio in *IGP–M* and the liability indexed to *IPCA* and conversely, in scenario 4, the asset portfolio is allocated in *NTN–B* and the index of liabilities is *IGP–M*. With these scenarios, we can analyze the effect of the mismatching of indexes on market risk capital based. In addition to the factors of interest rate risk, the mismatch between indexes may represent a significant impact on the market risk capital, specially in the Brazilian environment, in which both interest rate and inflation levels can be high. Finally, two additional scenarios were considered, taking into account a mixed asset portfolio, comprised by 50% of *NTN–B* and 50% of *NTN–C*. In scenario 5, the liability is fully indexed by the *IPCA* and, in scenario 6, fully indexed by the *IGP–M*.

Graph 1 indicates that the cash flows from liabilities have a decreasing trend over the years, since participants in the pension product, of varying ages, decease and no longer are entitled to the benefit. In this case, the actuarial calculation considers only the exit of the participants caused by the event associated with death and the amount of liabilities decreases until it becomes null. In contrast, the cash flows of assets invested in government bonds, represented in scenario 1 by *NTN–Bs*, has a very different behavior from the liabilities cash flows. This feature characterizes the existence of market risk associated with interest rates. It is important to note that the *LFT* cash flows are not considered. Due to its daily liquidity and adjustment by the *SELIC*, *LFT* can be interpreted as available cash that is used to pay very short term benefits.

Market risk should not only capture the mismatch of indexes, but also the mismatch of cash flows. Under scenario 1, the insurer would have a considerable need for risk capital based on market risk, given the high mismatch between inflows and outflows, especially in the long run. When maturity of cash inflows and outflows has a relevant discrepancy, which may be aggravated by possible mismatch of indexes, the nominal difference between the assets and liabilities may be higher than the real difference of the same flows. The cash balance, represented in the model by the *LFTs*, is used to remedy possible mismatches between the liability flows and the assets flows, when the liability is greater than the asset in a given period.

The cash flows depicted in Graph 1 are consolidated and the net present value of the gap is calculated, for all the 10, 000 simulations performed for each scenario in Table 2. Therefore, the simulations generate a frequency distribution. One can define risk capital based on the market risk of the insurance company using the concept of Value-at-Risk (*VaR*). To illustrate the procedure, we present the distribution of simulated present values, for scenario 1 described in Table 2, in Fig. 2.

Capital based on market risk is defined as the minimum amount value that is sufficient to cover 99.5% of potential losses arising from interest rate exposure. In scenario 1, the risk capital obtained is equivalent to *BRL* 12, 851, 412. As one of the objectives of the present study is to compare the results of the stochastic model with those of the Brazilian regulatory model, we also performed the calculation of market risk capital, following *CNSP* Resolution 321/2015. In the regulatory model, cash flows are deterministic and allocated in vertexes, with weighting factors defined in the resolution. The results of the allocation of the cash flows of assets and liabilities using the regulatory model for scenario 1 are presented in Fig. 3.

The analysis of the results suggests that, in the long run, even with asset cash flows ending earlier than liability's cash flows, there is still a cash balance to cover the benefit payments. However, this situation aggravates the need not only for the insurer to have high market risk capital but also to have effective asset management governance. Using the parameters established by the regulatory agency, risk capital based on market risk was only *BRL* 4, 426, 622. The value of the risk capital established in the stochastic model is, therefore, about 190% higher than the value obtained in the regulatory model, evidencing substantial differences. Although it is not

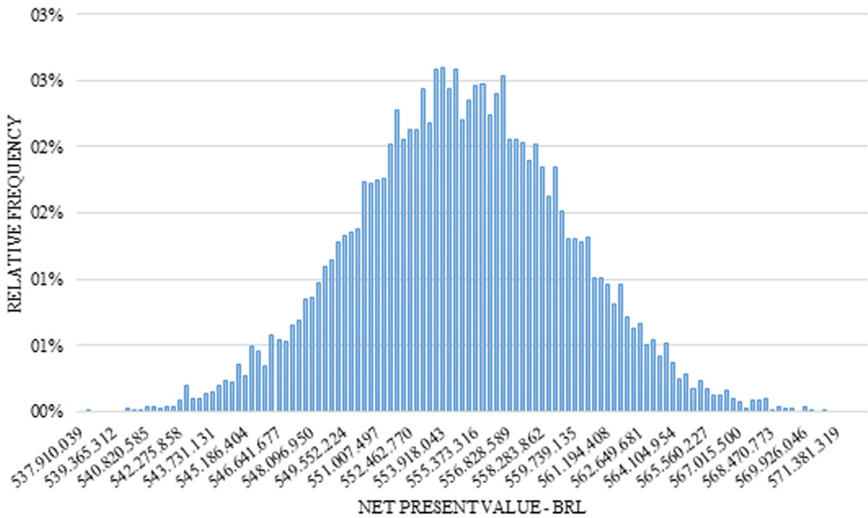

Fig. 2. Distribution of the presente value for scenario 1 using the stochastic model.

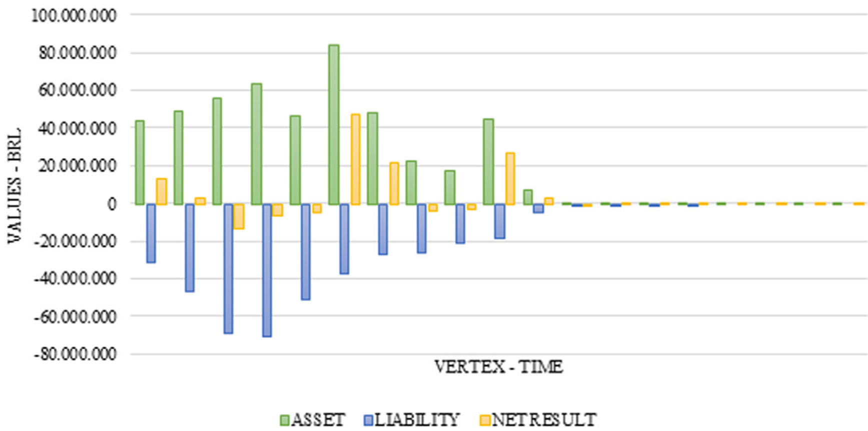

Fig. 3. Mismatch of cash flow for scenario 1 using the regulatory model.

possible to establish which capital requirement would be the closest to the ideal, the stochastic model can generate more direct information for the risk management process, since it allows to reflect current market parameters and leads to a frequency distribution.

The high discrepancy of results derives from the fact that, in the stochastic model, the use of probabilistic models for simulations of possible economic and demographic scenarios allows the capture of extreme conditions, with reflect on higher market risk capital requirements. Combinations of variables can generate a situation that represents an extreme impact on the solvency of the insurance company and, to a certain extent, the calculation of capital requirement based on a stochastic model can incorporate worst-case information. Another element that influences the difference between results from the stochastic and the regulatory models is related to the weighting factors of the flows in each vertex. Given the regulator's discretion in defining these factors for the industry as a whole, the adequacy to the exposures of the liabilities and assets of a particular insurer may be compromised. Characteristics of cash flows gaps and mismatches between indexes, as well as specific demographics of insurance product, may make the use of standard weighting factors, a particular historical series of prices and interests by the regulator, less adherent.

Finally, the same calculation process was carried out for the other proposed scenarios, aiming to analyze the behavior and the need for market risk capital. The comparative results obtained by the analysis of the present value of assets and liabilities for each scenario are presented in Table 7.

We can identify that the stochastic model leads to values that are substantially higher than those of the regulatory model, with variations from 15% in scenario 4 to 419% in scenario 2. It is shown that the scenarios, in which the regulatory model has grater distance from the stochastic model, are precisely the cases where the index of assets and liabilities is the same. In scenarios 3 and 4, it is possible to verify that the regulatory model has a greater focus on index mismatch sensitivity than on the stress of risk factors, since the mismatch provoked in these scenarios implied a large increase in market risk capital.

Considering that the stochastic model makes it possible to evaluate potential random situations, from the assets and liabilities characteristics of the analyzed portfolio, losses higher than those estimated by the regulatory model may be significant, making the

Table 7
Comparative results from required capital.

Scenario	Index		Required capital (BRL)		Difference (%)
	Asset	Liability	Stochastic model	Regulatory model	
1	IPCA	IPCA	12,415,886	4,426,622	120,18
2	IGP–M	IGP–M	25,711,442	4,954,327	418,97
3	IGP–M	IPCA	19,357,750	15,775,688	22.71
4	IPCA	IGP–M	16,686,035	14,469,940	15.32
5	50% IPCA 50% IGP–M	IPCA	15,581,574	7,993,907	94.92
6	50% IPCA 50% IGP–M	IGP-M	21,173,307	7,380,233	186.89

required capital insufficient to avoid financial problems for the insurer. This vulnerability can be further aggravated, as this study focused only on impacts from demographic and interest rate risks. Other market risk factors, such as correlation risk, were not analyzed, which could generate even more discrepancy between the values generated by the simulations and the regulatory model. The results suggest, therefore, the fragility of the regulatory model, since it does not capture possible fluctuations of the risk factors, which can lead to a relevant impact on the value of the market risk based capital.

We also analyze sensitivity of potential losses to shocks in interest rates of market risk factors as robustness checks to the results. The model used to measure risk capital based on market risk considers the present value of the cash flows of assets and liabilities. Therefore, a change in the values of the variables in the forecast and discount of cash flows impacts gaps in assets and liabilities, which can define new risk capital requirements based on market risk.

We present results from the sensitivity analyzes in Tables 8 and 9, showing that changes in the parameters of the model result in different need for capital compared to the baseline scenario of *BRL* 12,915,886, depicted in Table 7. In Table 8, the sensitivity scenarios numbered from 1 to 10 were generated by increasing and decreasing interest rates, from 100 to 500 bps in the SELIC and *NTN-B* term structures. Increase in interest rates, in scenarios 1 to 5, results in a reduction in the present value of cash flows and, consequently, in a decrease in the volume of risk capital based on market risk. Scenarios 6 to 10 lead to an opposite effect, and the decrease in interest rate implies higher present value of cash flows and a higher risk capital. In scenarios 11 to 14 of Table 8 and scenarios 1 to 4 of Table 9, we conducted a sensitivity analysis related to inflation, which is relevant in Brazil. These inflation indexes have effects on both the asset and liability sides of the balance sheet of insurance companies.

In scenarios that simulate increasing inflation indexes, nominal values of cash flows as well as capital requirements will be higher. In contrast, declining inflation will result in less capital. Therefore, price stability can be beneficial to insurance companies regarding capital requirements. Scenarios 15 and 16 in Table 9 show results for shocks in both interest rates and inflation. Results from sensitivity analysis show that differences in capital charges are in the range between -9.48% and 13.24%, for a maximum fluctuation of 500 bps in interest rates, as shown in Tables 8 and 9. It is important to consider that results are also sensitive to maturity gaps between asset and liabilities cash flows. The lower the gaps, the lower the sensitivity of capital to model parameters. Results reinforce that stochastic models, subject to shocks in market risk factors, lead to different capital requirements than deterministic models.

In addition, it is important to highlight, although that not on the scope of this study, that *SUSEP* has already defined mechanisms for calculating operational, credit, and underwriting risks for the insurance market. These mechanisms follow a method similar to market risk, considering deterministic models dependent on weighting factors established by the regulator.

Given the use of standardized models to determine the required minimum capital, the specific characteristics of each insurance company are not considered in regulatory models, making the results less useful to managers. In contrast, stochastic models that explore parameterized simulations using characteristics of the insurer's portfolio and market data, give more comprehensive

Table 8
Sensitivity of results to model parameters of the term structure and the IPCA inflation index.

Scenario	Bps	Changed risk factor	Capital	Change to baseline
1	100	SELIC and NTN-B	12,200,644	−1.73%
2	200	SELIC and NTN-B	11,807,626	−4.90%
3	300	SELIC and NTN-B	11,552,684	−6.95%
4	400	SELIC and NTN-B	11,238,879	−9.48%
5	500	SELIC and NTN-B	11,346,876	−8.61%
6	−100	SELIC and NTN-B	12,840,472	3.42%
7	−200	SELIC and NTN-B	13,205,341	6.36%
8	−300	SELIC and NTN-B	13,715,980	10.47%
9	−400	SELIC and NTN-B	14,059,257	13.24%
10	−500	SELIC and NTN-B	13,764,208	10.86%
11	50	IPCA	12,605,577	1.53%
13	−50	IPCA	12,409,486	−0.05%
14	−100	IPCA	12,385,319	−0.25%
15	300 / 100	SELIC and NTN-B / IPCA	11,989,184	−3.44%
16	3	SELIC and NTN-B / IPCA	13,317,225	7.26%

Table 9
Sensitivity of results to model parameters of the IGP-M inflation index.

Scenario	Bps	Changed risk factor	Capital	Change to baseline
1	100	IGP-M	26,619,096	3.53%
2	200	IGP-M	27,449,471	6.76%
3	−100	IGP-M	25,515,961	−0.76%
4	−200	IGP-M	25,198,733	−1.99%

information for the decision-making process. Obtaining a probability distribution of present values of assets and liabilities allows not only the definition of mean values but also values of loss percentiles. This additional information is useful in defining risk capital for regulatory purposes, and other metrics such as value-at-risk, which can be used for effective risk management.

5. Final considerations

The implementation of *SUSEP*'s regulatory capital requirement provides advances towards greater risk management governance for insurance companies operating in Brazil. This capital requirement seeks to safeguard investors and insurance clients against sudden impacts on the values of assets and liabilities. However, the model established by the Brazilian regulatory agency has substantial limitations, mainly because it takes into account a deterministic calculation method, which risk factors are defined by the regulator itself, without necessarily reflecting the risk of the insurer's portfolio or providing a more immediate update to changes of market conditions.

It is important to highlight that comparison between deterministic and stochastic models is a relevant topic. For instance, Loisel and Milhaud (2011) study surrender risk models for life insurance contracts. The authors confront a stochastic model of the surrender rate with a traditional deterministic S-shaped curve.

In this context, this study discusses a stochastic market risk capital model, taking into account random aspects of the actuarial liabilities, with estimates of parameters based on a real portfolio of a Brazilian insurance company. Cash flows were discounted to present value by interest rate structures using diffusion processes with mean reversion. Simulations of random variables associated with survival of participants and interest rates were conduct to determine risk capital based on the market risk of an insurance company. The results from the stochastic model are comparing with the requirement defined by the Brazilian regulatory agency. In all the distinct scenarios of mismatch between asset and liability indexes, the fragility of the regulatory model is evident. Results obtained by the stochastic model were substantially higher than the results obtained by the regulatory model. The study suggests that the deterministic method used in the regulatory model does not capture possible extreme scenarios, which is fundamental when trying to measure the risk capital of an insurance company.

Insurance companies are required to follow the regulatory model. The expectation is that, in the medium term, *SUSEP* will allow insurance companies to estimate risk capital based on internal models. We highlight that the implementation of a stochastic internal model can provide important information for the decision-making process and the management of risks of an insurer. In the Brazilian case, however, the development of internal models is still a challenge, given the requirement of a highly qualified team, adequate software resources and data availability.

The objective of the study was to analyze a stochastic model that, focusing only on market risk, illustrates possible fragilities of the Brazilian regulatory model. Despite the limitations of the study, such as analyzing a portfolio with simple exposures to demographic characteristics and interest rate risk, the study allows to alert managers of insurance companies that the regulatory model should not replace a managerial model that could better fit the specific characteristics of the insurer's portfolio.

In our study, using actual actuarial liability data and a simulated investment portfolio that reflects traditional investments of insurance companies, we found that probabilistic approach could lead to estimates of potential loss that are four times higher than values using the deterministic regulatory model in the Brazilian insurance industry.

Therefore, our study not only strengthens the argument that deterministic models are underestimating risks, but also that differences can be significant. In this context, the study contributes to the insurance regulation by providing arguments to the need of more realistic models than the deterministic approach currently established by regulatory agencies in Brazil. More specifically, stochastic models can improve measurement of risk exposure and promote a more resilient insurance industry.

Our study alerts to the need for higher capital requirements. It also brings reflections for regulators and managers in the Brazilian insurance industry. Although results point out to an underestimated risk exposure, regulators cannot simply impose new models that imply substantially higher capital charges. The impact on the sector could be considerable, even jeopardizing many operations.

In Brazil, interest rates are depicting a declining trend, impacting the returns of the investment portfolio of insurance companies. At the same time, higher longevity levels may lead to a reassessment of actuarial liability. Therefore, sudden adjustments in capital requirements due to an eventual adoption of stochastic models may lead to instability in the sector.

Nevertheless, if potential loss estimates are aggressive when using deterministic models, it is inevitable that more capital will be required in the future. In this context, managers of insurance companies should anticipate regulation and prepare themselves for lower leverage levels, with significant impacts in asset and liability management.

In an environment of declining interest rates and raising actuarial liability, asset allocation can be challenging. Managers of insurance companies will have to enhance operational efficiency and simultaneously look for more profitable investments.

It is important to highlight that the very search for investments with higher returns to face increasing liabilities can also amplify market risk as managers will have to increase the weight of volatile assets in their asset portfolios. Taking into account this prospective scenario, insurance companies in Brazil will have to endure difficult obstacles. By implementing internal risk management models (e.g. based on stochastic process as the one discussed in this study), insurance companies may develop expertise and prepare themselves for future changes in regulation.

The paper confronted a regulatory deterministic model and a stochastic model for capital requirements of market risk. We also conducted a sensitivity analysis by simulating shocks in market risk factors. Further studies could compare sensitivity of results to different interest rate models (e.g. Vasicek (1977); Hull and White (1990); Nelson and Siegel (1987); Svensson (1994), etc).

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